

ESF project

“Establishment of interdisciplinary scientist group and modelling system for groundwater research”

# Mathematical model of Baltic artesian basin

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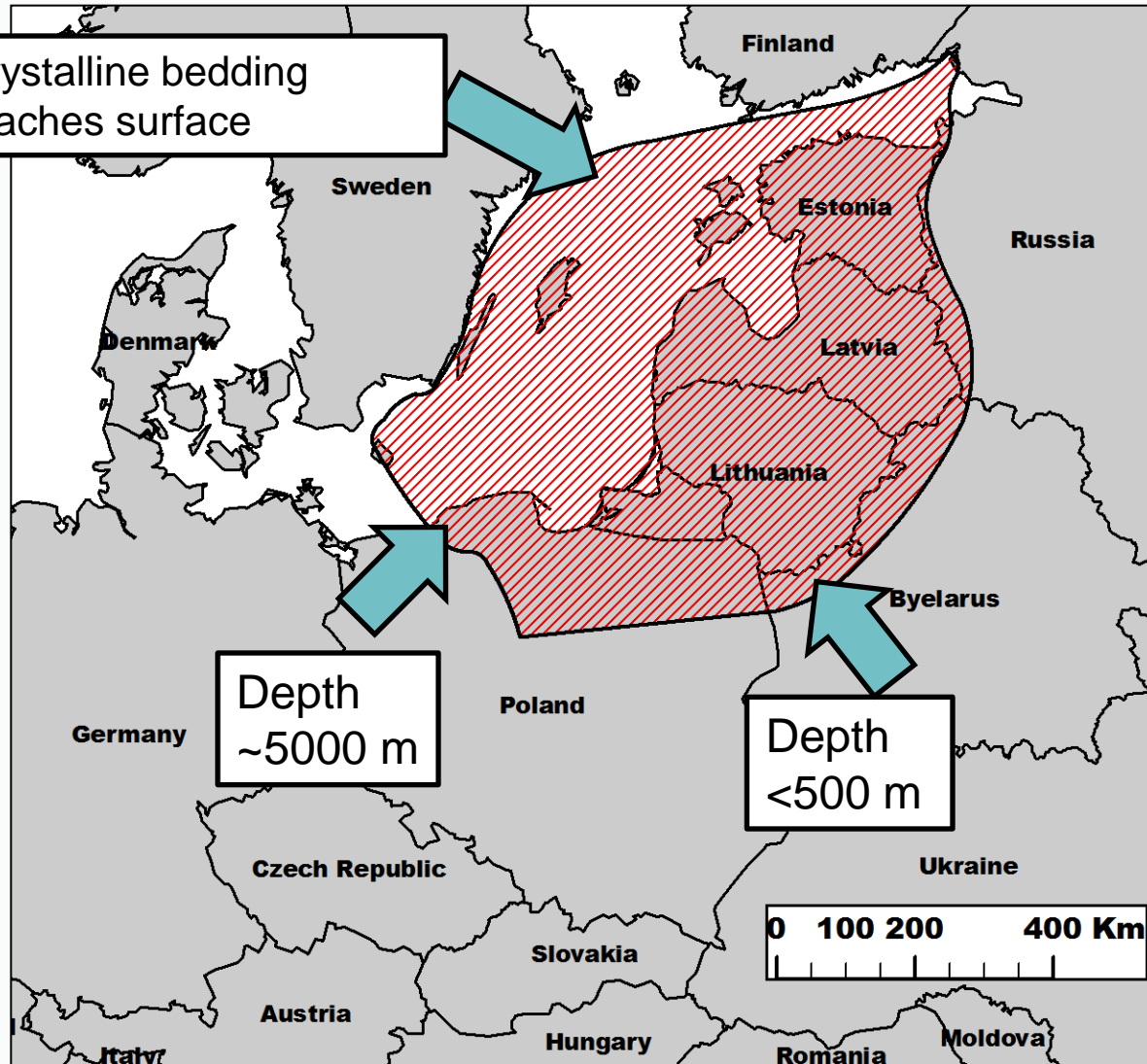
## Motivation

There exist several local modelling studies of ground water flow for the parts of the Baltic artesian basin (BAB)

The aim of the present work is the development of a closed hydrogeological mathematical model of the whole BAB

# Area of study

Crystalline bedding reaches surface



Depth  
~5000 m

Depth  
<500 m



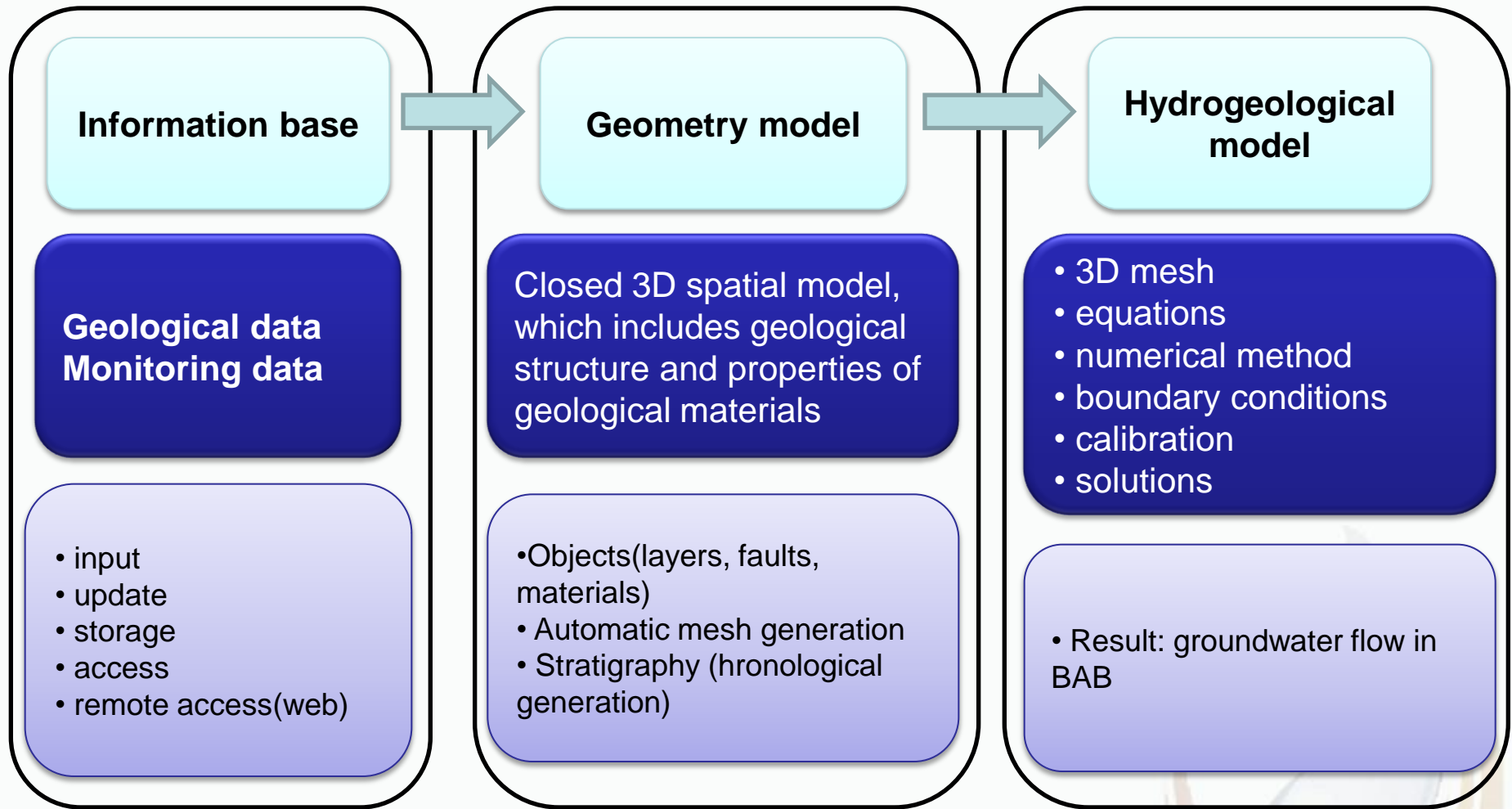
Baltic artesian basin (BAB) is a multi-layered and complex hydrogeological system up to 5000 m deep

BAB fully covers the territory of Latvia, Lithuania and Estonia, parts of Poland, Russia, Belarus as well as large area of the Baltic Sea, including island of Gotland.

It's area is approximately 480000 km<sup>2</sup>

It is the main drinking water source in the Baltic countries

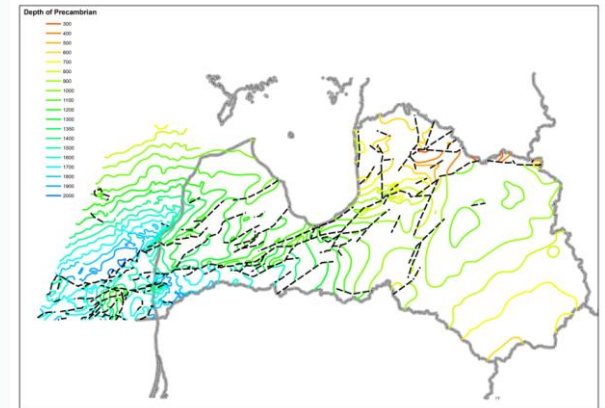
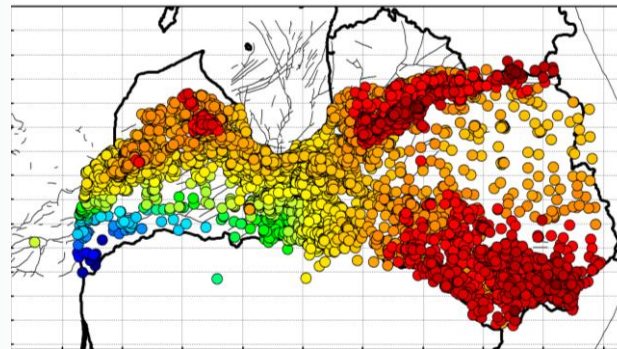
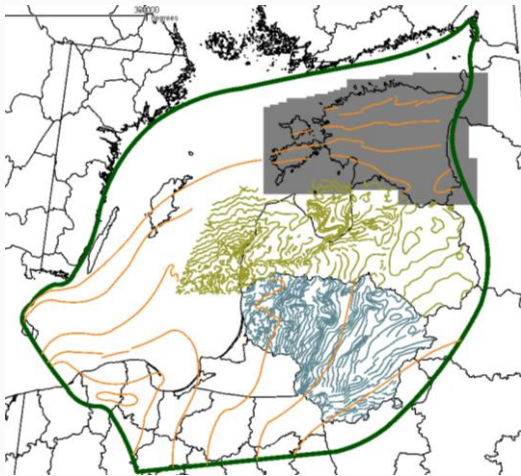
# Scheme of integrated model system development



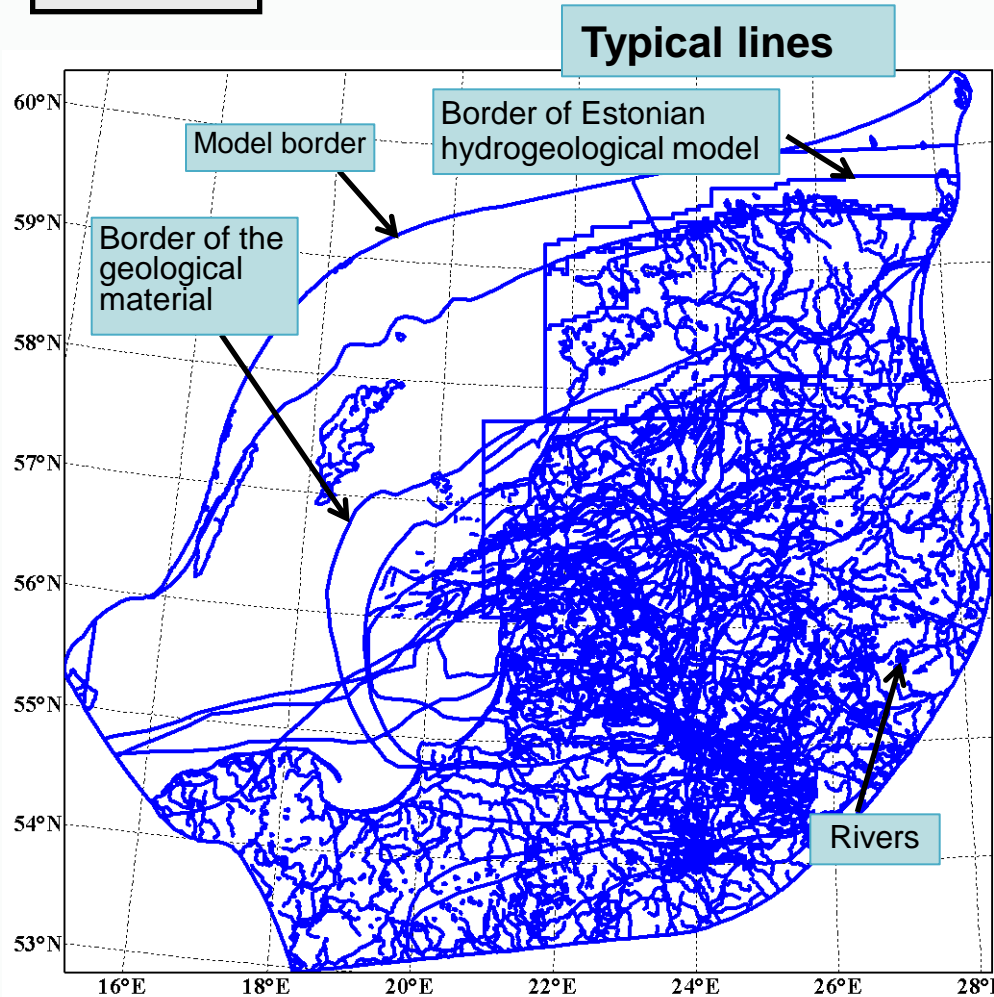
# Data sources

It is a challenge to unify the heterogeneous information from different sources, which are employed for building of the geometrical structure of the model. Algorithms are developed for this purpose considering the priority, importance and plausibility of each data source

- (1) Maps of height isolines of geological layers for Latvia and Lithuania
- (2) Maps of sub-quaternary deposits in Latvia and Lithuania
- (3) Maps of fault lines on the basement surface
- (4) Stratigraphic information for the Latvian boreholes (around 20'000)
- (5) Layers from Estonian hydrogeological model
- (6) Earth topography
- (7) Depths of Baltic sea
- (8) Data from published geological cross-sections, information from books and other sources.



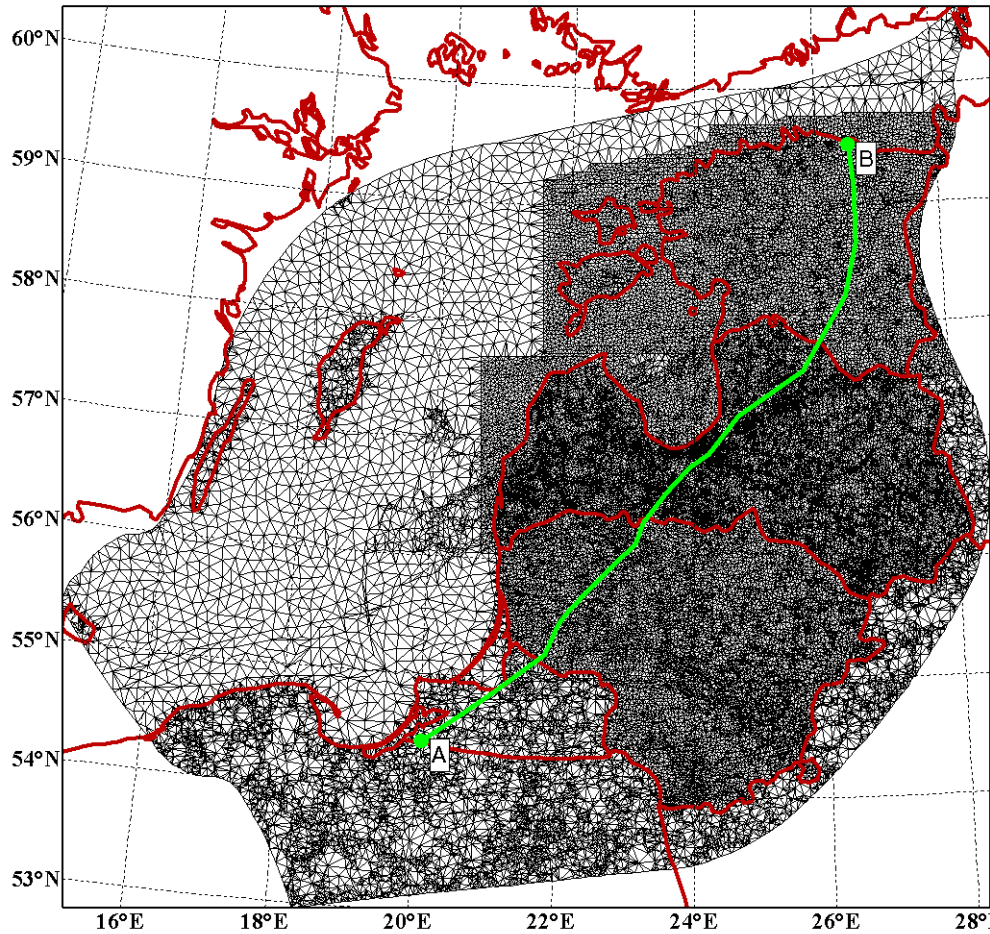
# Mesh



*Edges of triangular mesh coincide with the line data*

- Finite element (FE) method was employed for the calculation of the 3-dimensional groundwater flows with free surface.
- 3D mesh was constructed layer-wise.
- The triangular mesh in horizontal plane was constructed including characteristic lines such as rivers, borders of countries and areas of presence of geological layers.
- Fault lines are also taken into account considering the displacements along the fault
- Most of the 3D finite elements are triangular prisms. Pyramids and tetrahedra are used near the fault lines and wedge lines of geological layers.

# Mesh



*Finite element mesh, view from the top.*

*Higher resolution of mesh in areas with sufficient geological data*

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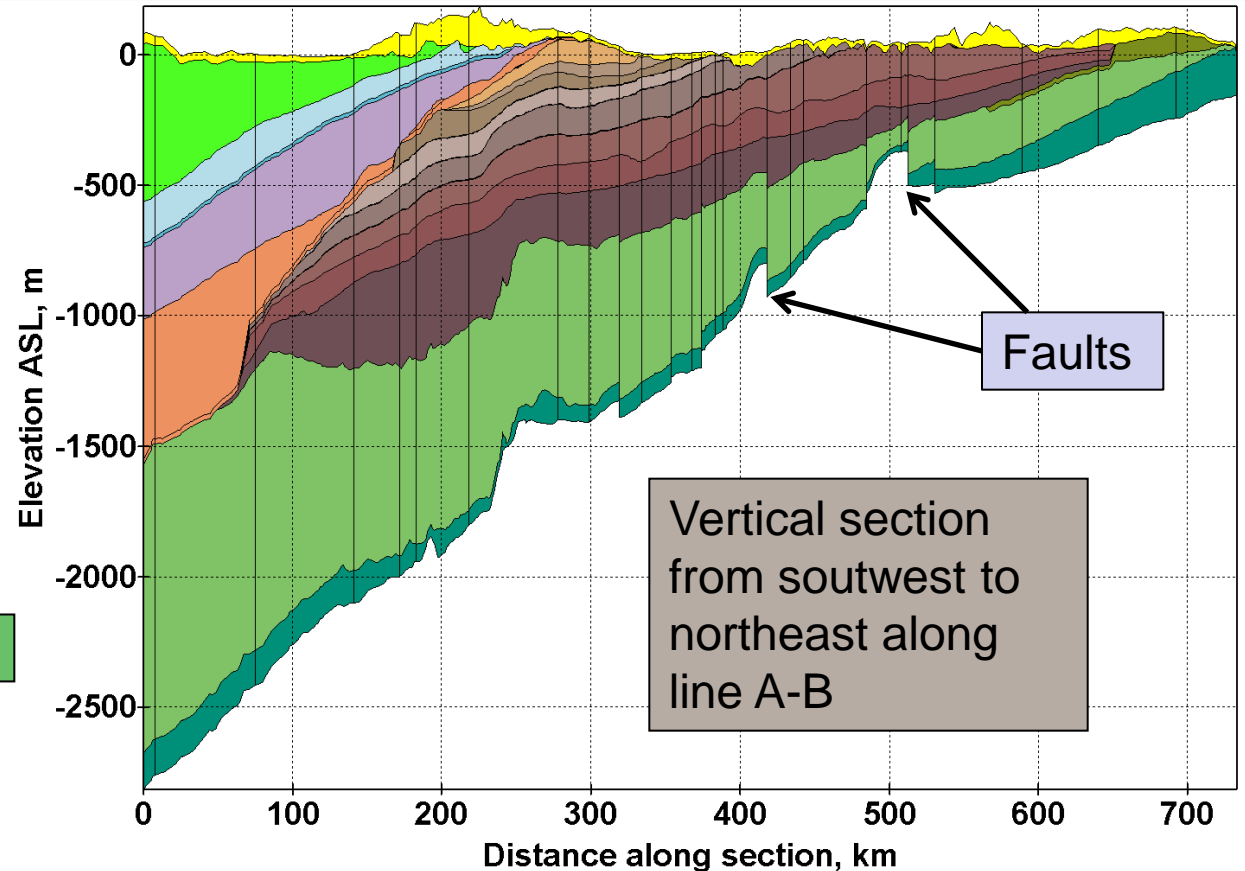
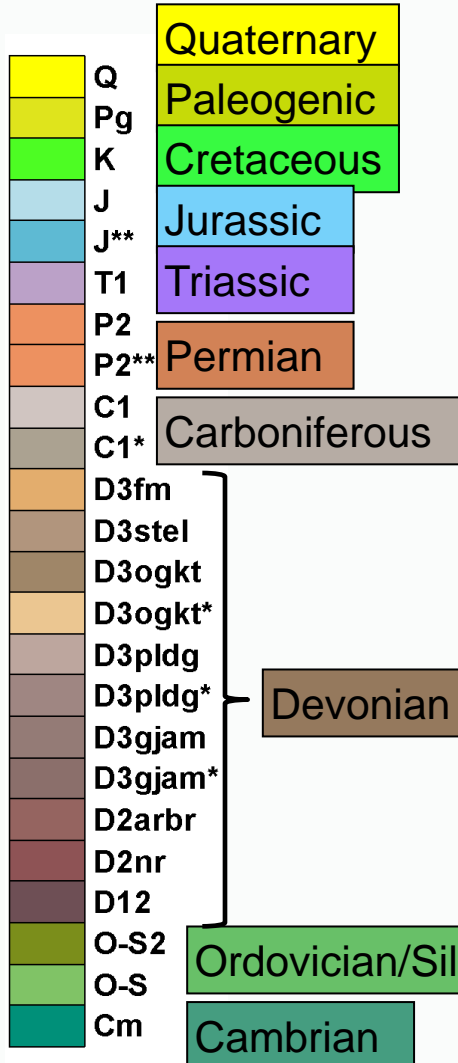
# Geological structure

Geological structure consists of 24 layers distinguished on the basis of each geological unit hydraulic properties and geological data resolution

In total 14 aquifers, and 10 aquicludes are present in the current version of the model

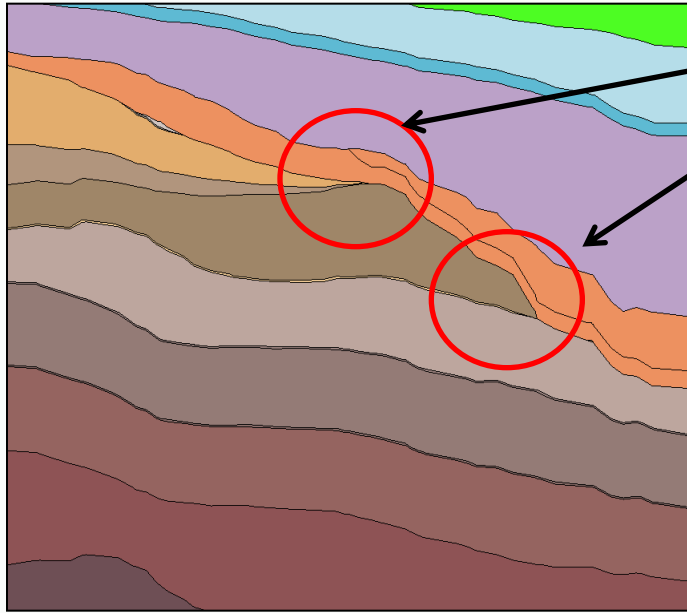
Each stratified layer is assumed to have similar hydraulic properties

Quaternary sequence is treated as a single layer with varied water permeability as deduced from the borehole data



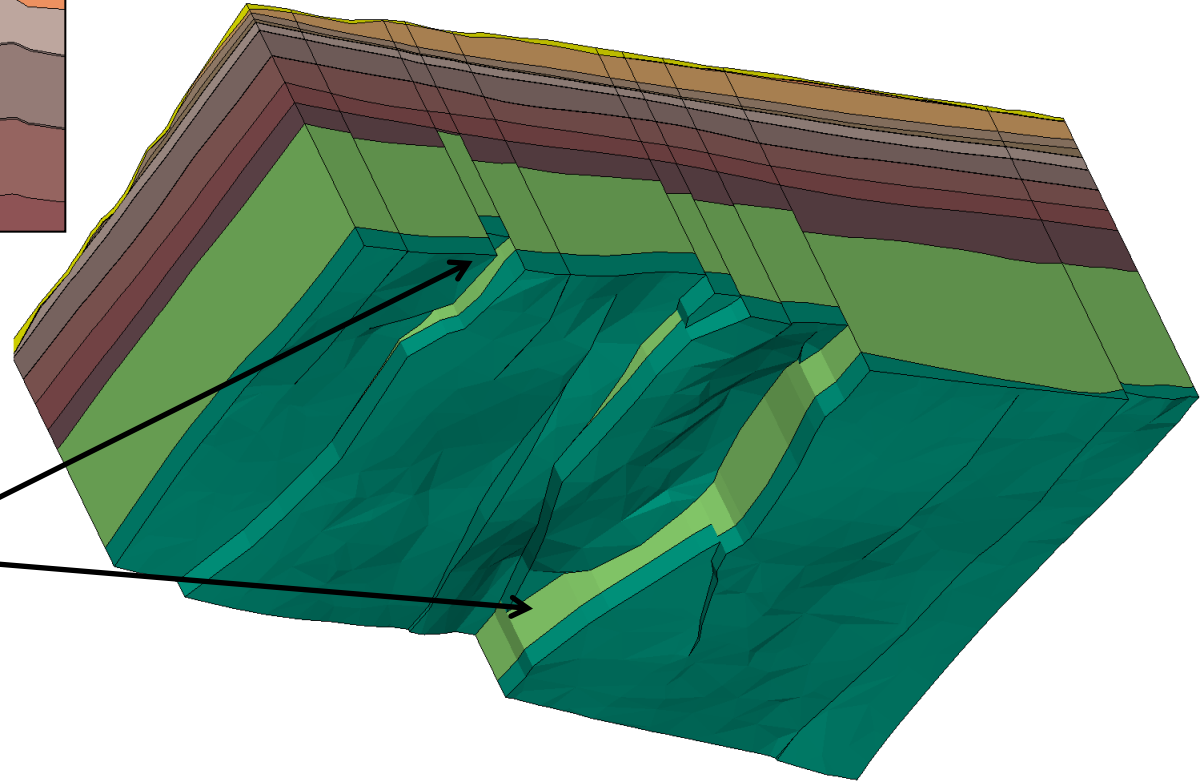


# Geometric model building, highlights



Wedging out of layers.  
*Pyramidal and tetraedral elements are used instead of triangular prisms*

Tectonic faults are included in mesh, taking into account (variable) vertical displacement.  
*Remesh of layers is necessary.*



# Calculation of groundwater flow

**Stationary** (*e.g. steady and constant on a longer time frame*) flow is calculated

## **Boundary conditions:**

- 1.As model covers all of the BAB area, no-flow conditions are defined for the side boundaries
- 2.No-flow boundary conditions were applied on the rock bottom
- 3.Infiltration conditions are applied on the surface (*as a result of simple hydrological model*)
- 4.Mean discharge values are applied for the water abstraction wells (*in places where data is available*)

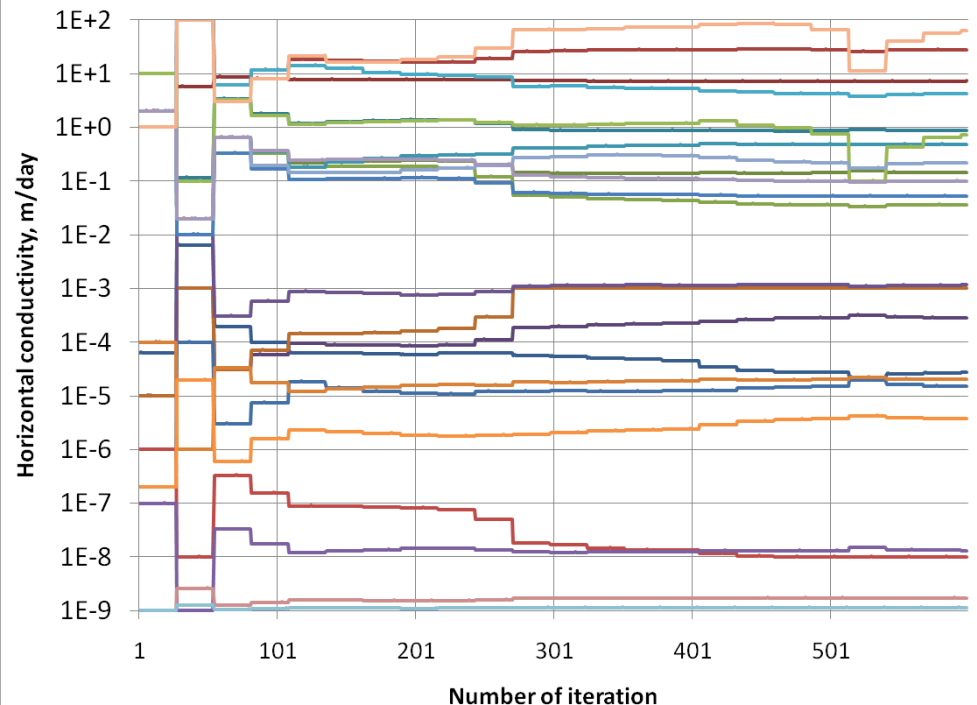
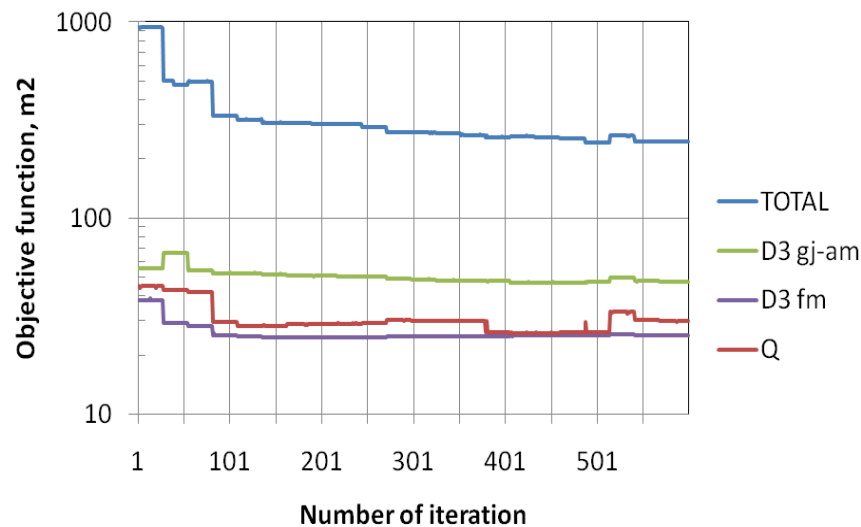
## **Material properties:**

- 1.Constant horizontal and vertical hydraulic conductivity values for each layer, determined during the calibration.
- 2.Quaternary– spatially variable conductivity. Conductivity is calculated using specially designed algorithm (in territory of Latvia).

**Calculation results** are piezometric head in each mesh point in each layer and flow velocity field (derived from the piezometric head field).

# Model calibration using optimization method

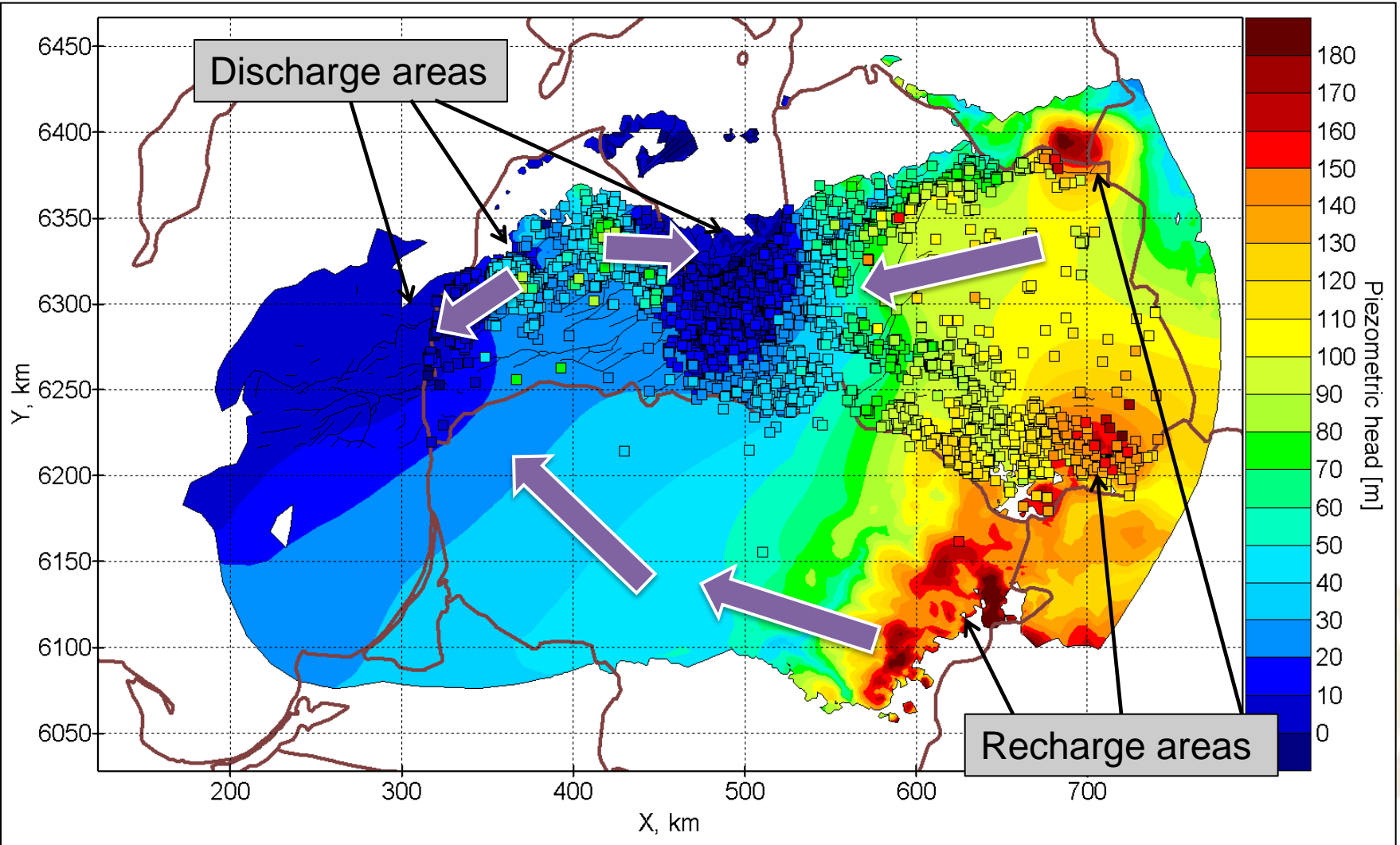
- Quasi-Newton optimization method L-BFGS-B
- Objective function is sum of weighted squared differences (observed-computed) of water level in all monitoring wells and boreholes
- Weight functions account for borehole density and time moment (monit. time - year 2000)
- The parameters of the calibration are the horizontal and vertical hydraulic conductivities of hydrogeological layers
- The allowed variation range of conductivities for all layers is by 0.01 till 100 times compared to original value



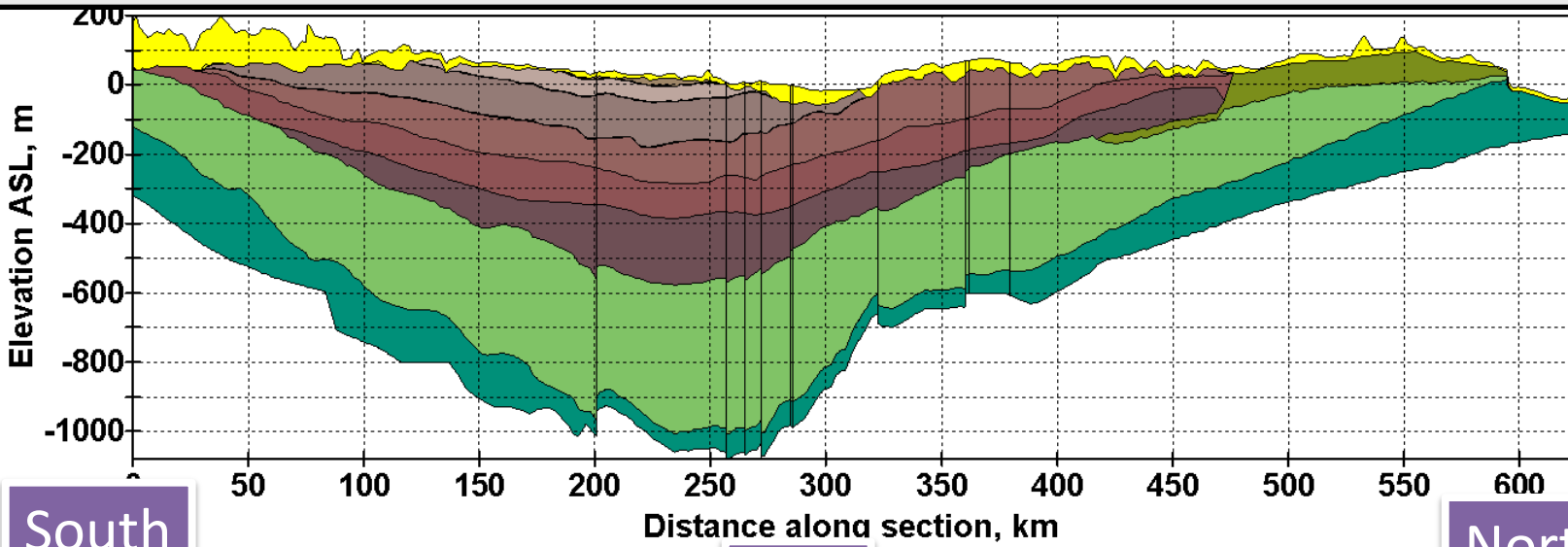
# Distribution of piezometric head in D3 gj-am layer

Points represent observed heads

Arrows show schematic dominant flow directions



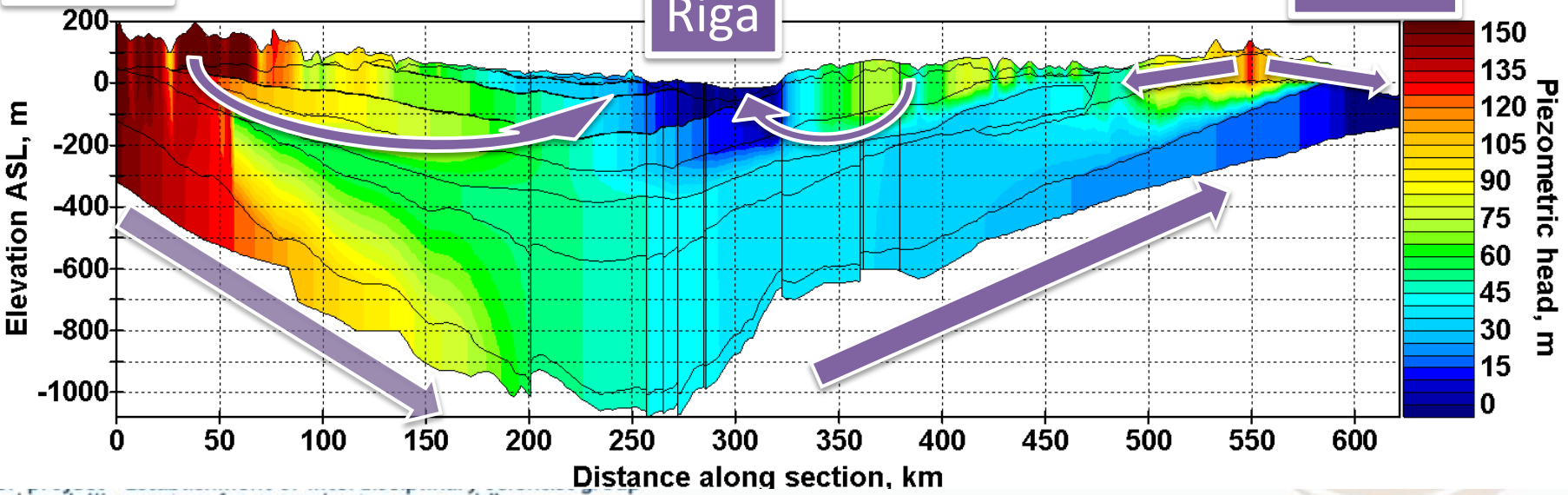
# Distribution of piezometric head in south-north vertical crosssection



South

North

Riga



# Summary

Data for the building of regional model of Baltic artesian basin has been collected and prepared

Geometry model of the Baltic artesian basin geological structure is developed, consisting of 24 layers

Steady-state hydrogeological model has been set-up and calibrated

The distribution of piezometric heads and principal flows inside BAB was analyzed based on the model results

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